

Hierarchical Reasoning Model



00 – Intro

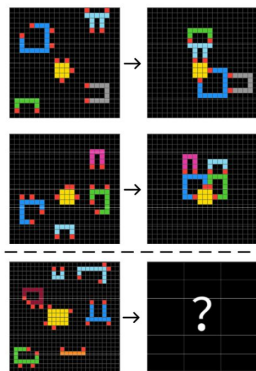
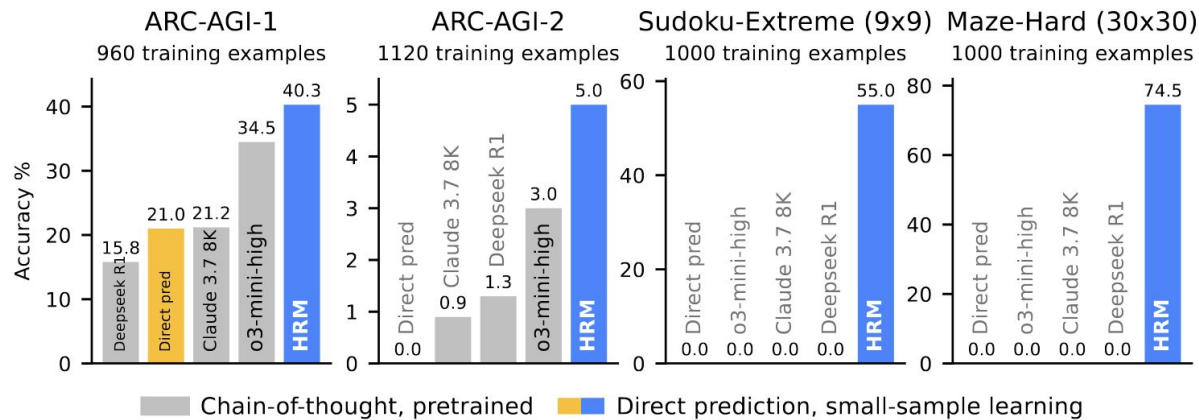
Tyler Crosse

About Me

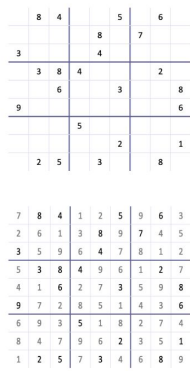
- MSCS at Georgia Tech focus on ML + Systems (Expected May 2026)
- 5 Years Knowable, Legal Contract Analytics
 - Data Ingestion & Processing
 - Information Retrieval & Presentation
- 2 Years Rosetta Stone
 - Experiment tracking and iteration
- BS Biomedical Engineering

<https://www.linkedin.com/in/tylercrosse>

01 – Paper + Model



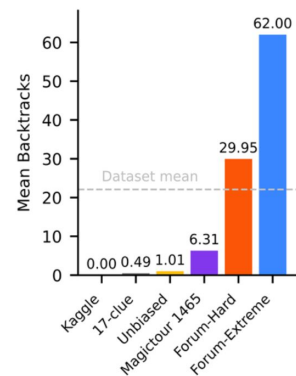
(a) ARC-AGI



(b) Sudoku-Hard

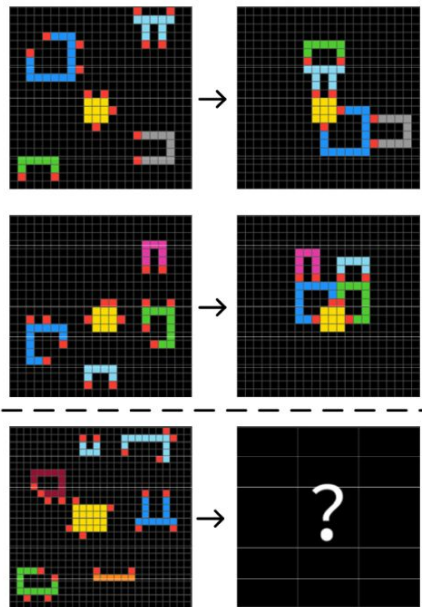


(c) Maze navigation



(d) *Sudoku-Extreme* subset difficulty

Figure 6: **Left:** Visualization of benchmark tasks. **Right:** Difficulty of *Sudoku-Extreme* examples.

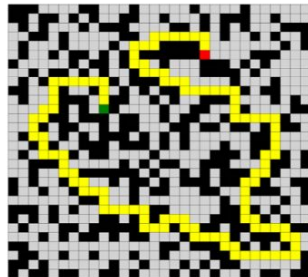
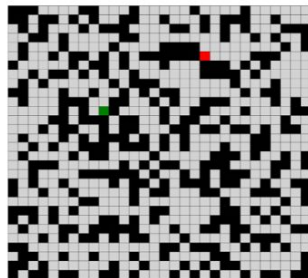


(a) ARC-AGI

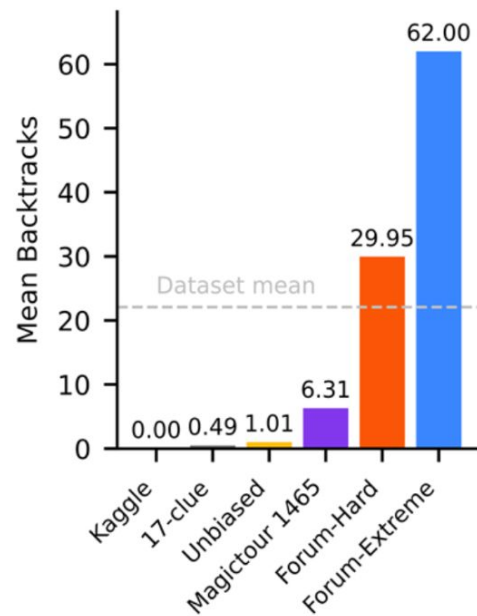
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7	8	4	1	2	5	9	6	3
2	6	1	3	8	9	7	4	5
3	5	9	6	4	7	8	1	2
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4	1	6	2	7	3	5	9	8
9	7	2	8	5	1	4	3	6
6	9	3	5	1	8	2	7	4
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1	2	5	7	3	4	6	8	9

(b) Sudoku-Hard



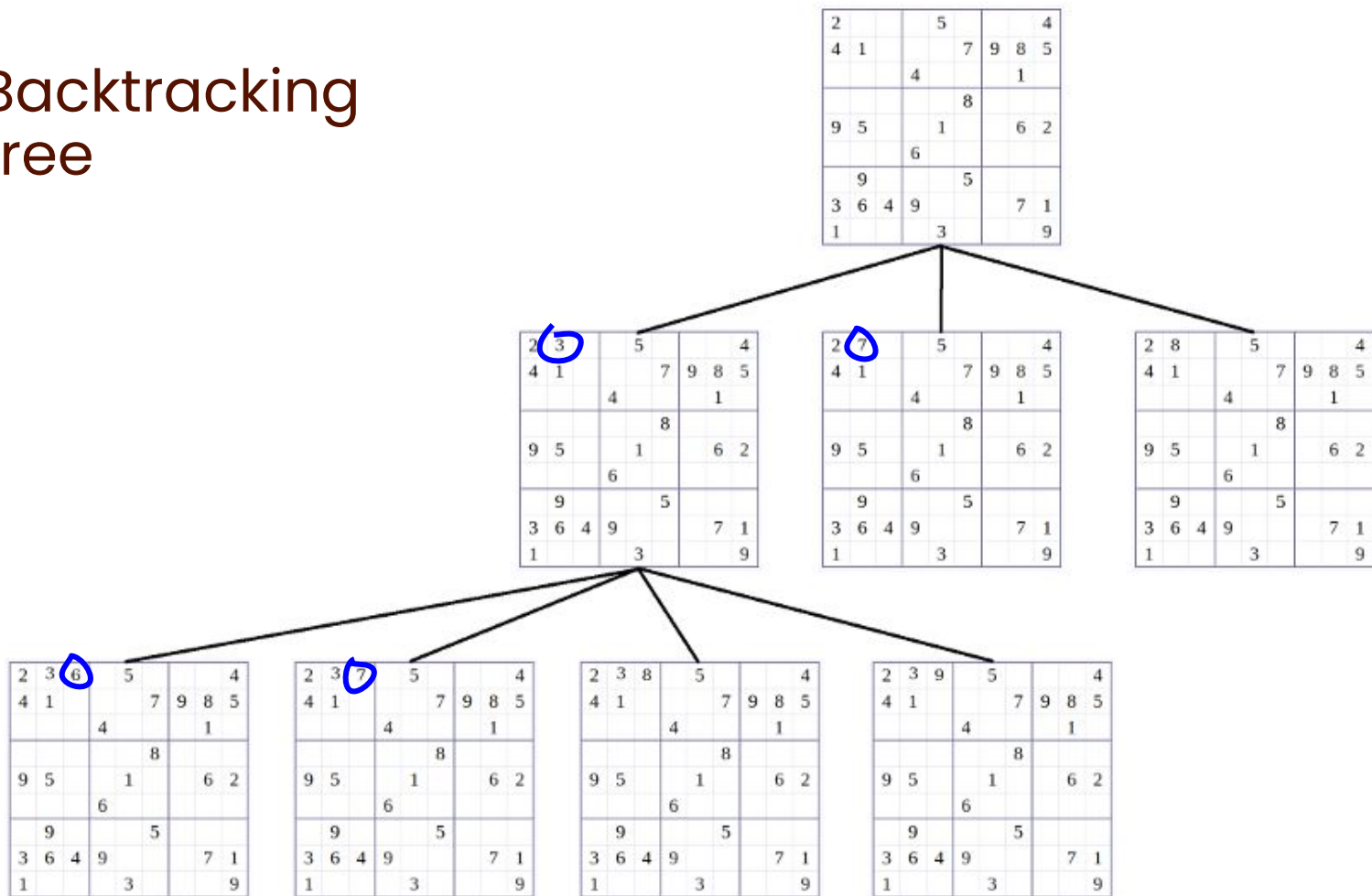
(c) Maze navigation



(d) *Sudoku-Extreme* subset difficulty

Figure 6: **Left:** Visualization of benchmark tasks. **Right:** Difficulty of *Sudoku-Extreme* examples.

Sudoku Backtracking Search Tree



Hierarchical Reasoning High (H) + Low (L)

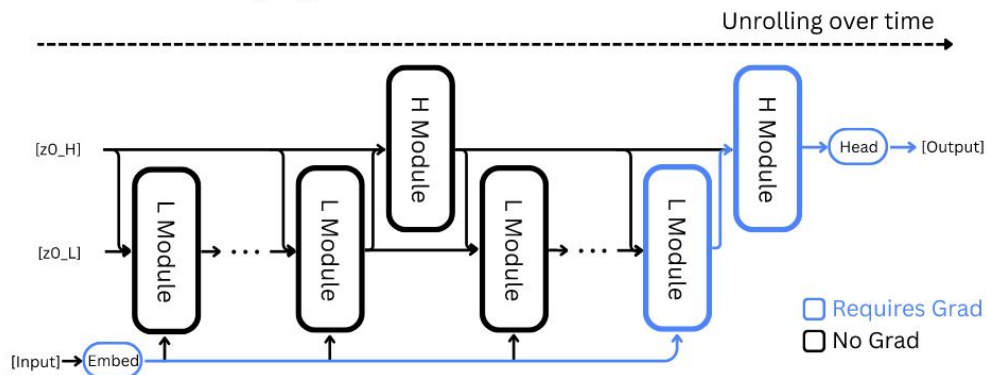


Fig 4a: Diagram of HRM with approximate gradient

```
def hrm(z, x, N=2, T=2):
    x = input_embedding(x)
    zH, zL = z

    with torch.no_grad():
        for _i in range(N * T - 1):
            zL = L_net(zL, zH, x)
            if (_i + 1) % T == 0:
                zH = H_net(zH, zL)

    # 1-step grad
    zL = L_net(zL, zH, x)
    zH = H_net(zH, zL)
    return (zH, zL), output_head(zH)

# Deep Supervision
for x, y_true in train_dataloader:
    z = z_init
    for step in range(N_supervision):
        z, y_hat = hrm(z, x)

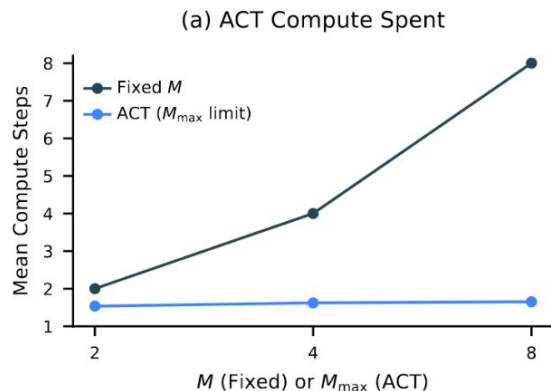
    loss = softmax_cross_entropy(y_hat, y_true)
    z = z.detach()

    loss.backward()
    opt.step()
    opt.zero_grad()
```

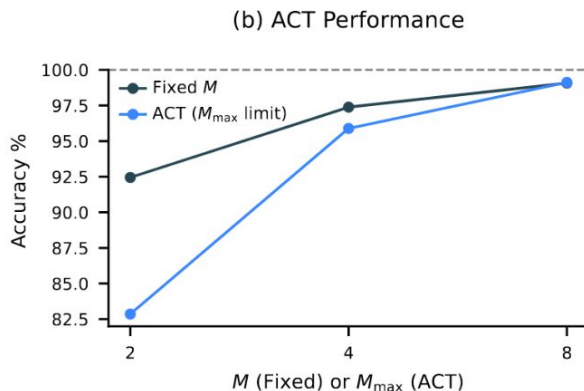
Fig 4b: Pseudocode of HRM with deep supervision



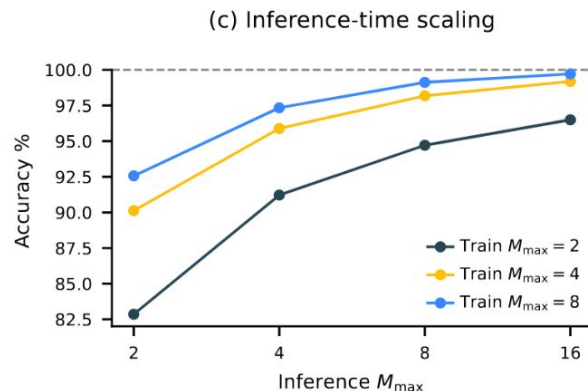
Adaptive computational time (ACT)



(a) Mean compute steps used by models with ACT versus models with a fixed number of compute steps (M). ACT maintains a low and stable number of average compute steps even as the maximum limit ($M_{\max}$) increases.

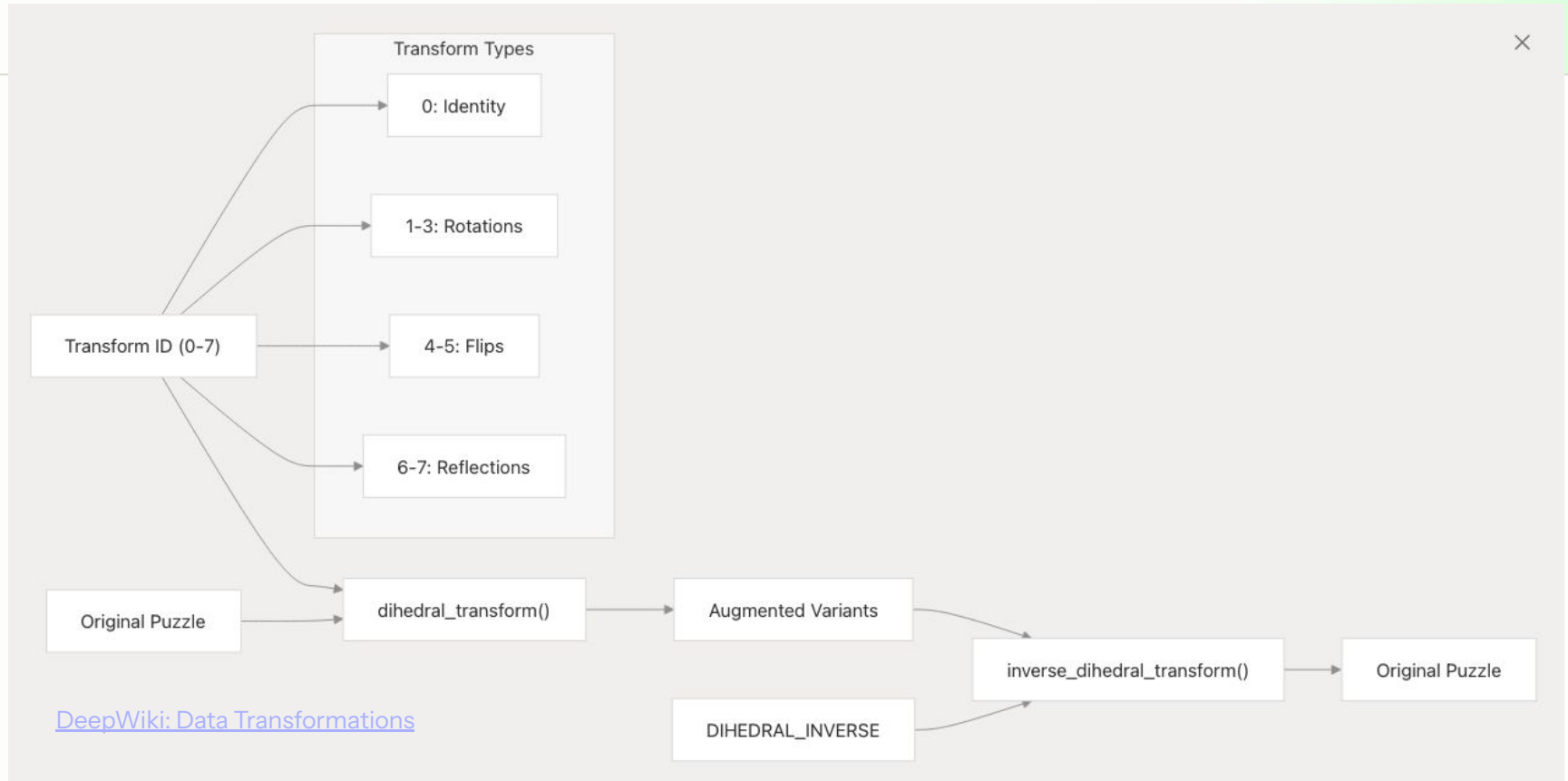


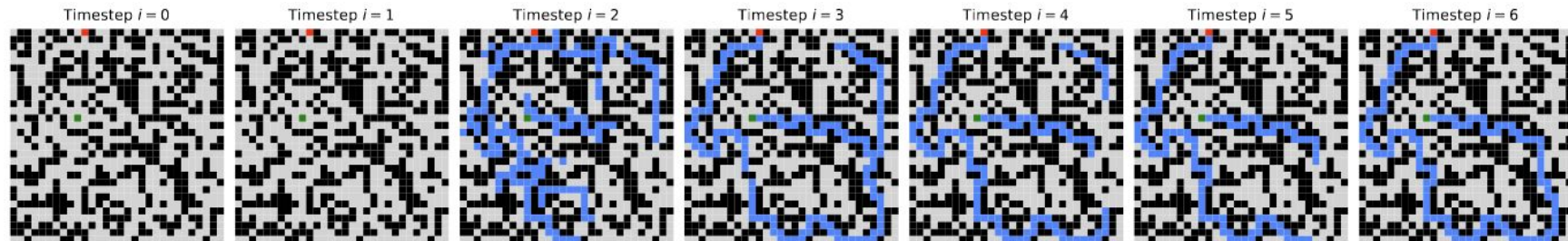
(b) Accuracy comparison. The ACT model achieves performance comparable to the fixed-compute model while utilizing substantially fewer computational steps on average.



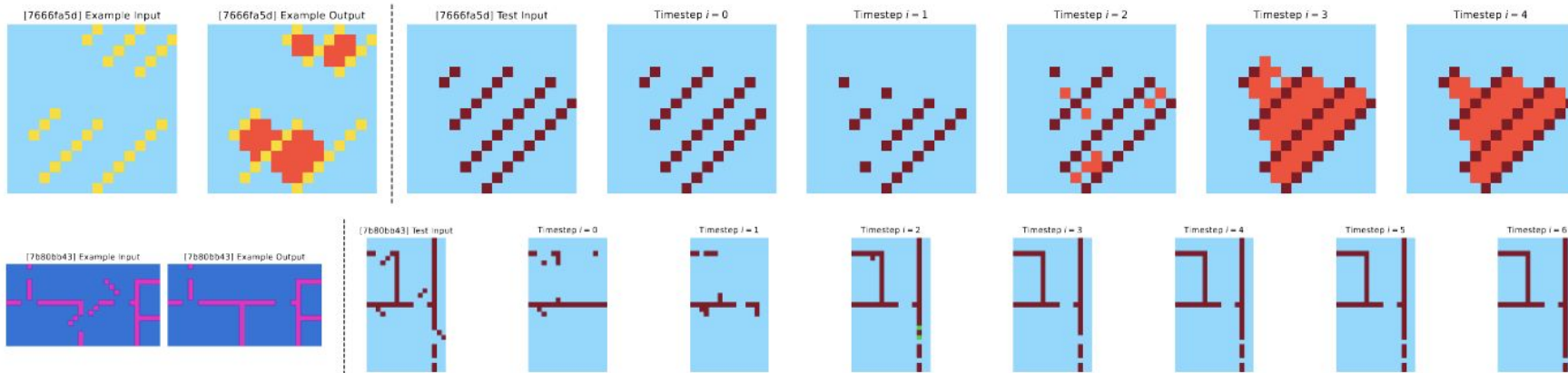
(c) Inference-time scalability. Models trained with a specific $M_{\max}$ can generalize to higher computational limits during inference, leading to improved accuracy. For example, a model trained with $M_{\max} = 8$ continues to see accuracy gains when run with $M_{\max} = 16$ during inference.

Data Augmentation





Initial	Timestep $i = 0$									Timestep $i = 1$									Timestep $i = 2$									Timestep $i = 3$									Timestep $i = 4$									Timestep $i = 5$									Timestep $i = 6$									Timestep $i = 7$																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
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02 – ARC Analysis

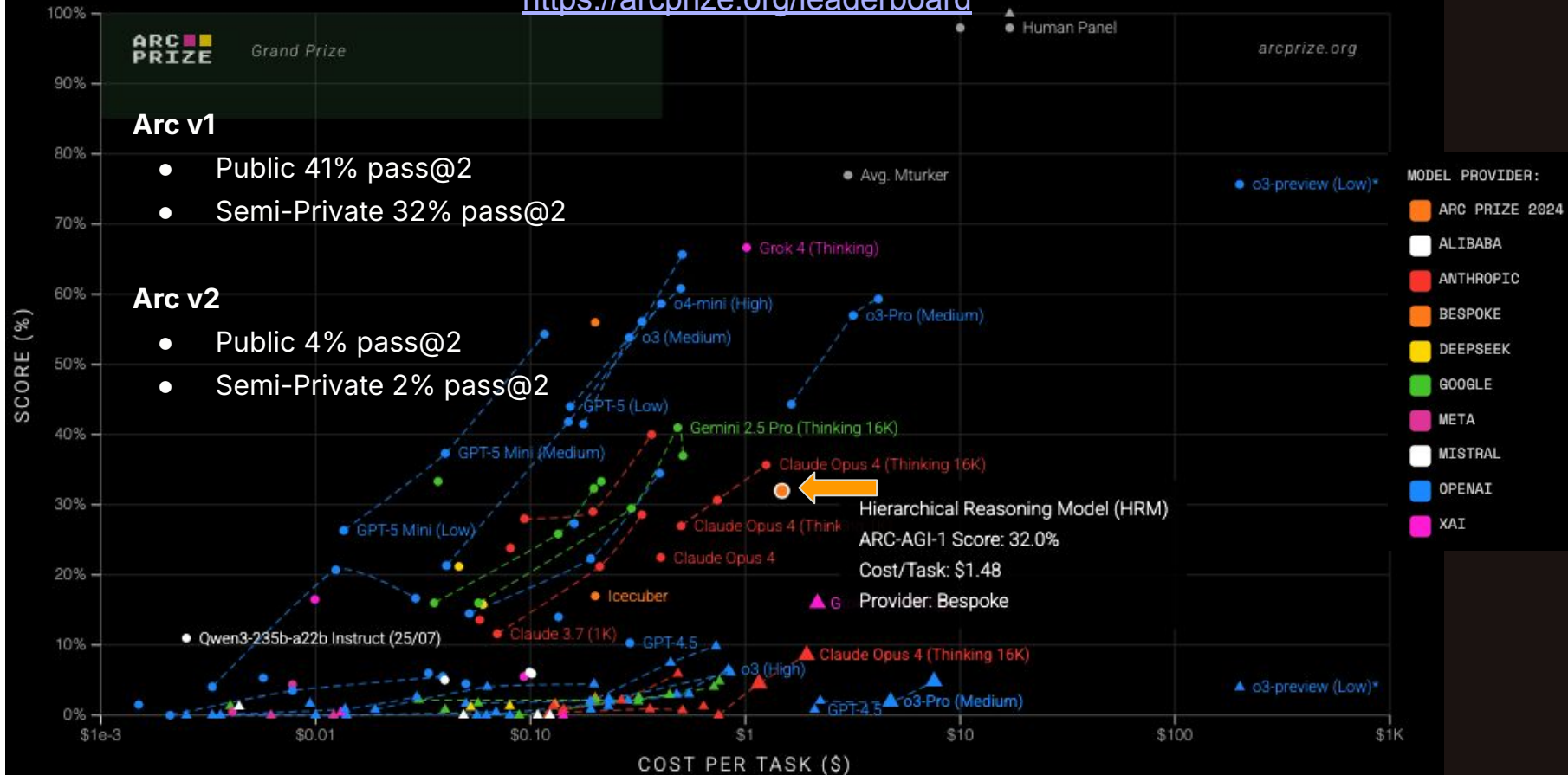
Why does it work?

Candidates

1. Model architecture
2. Inner refinement loop
3. Outer refinement loop, with or without ACT
4. Data augmentation for HRM
5. Puzzle embeddings

ARC-AGI LEADERBOARD

<https://arcprize.org/leaderboard>



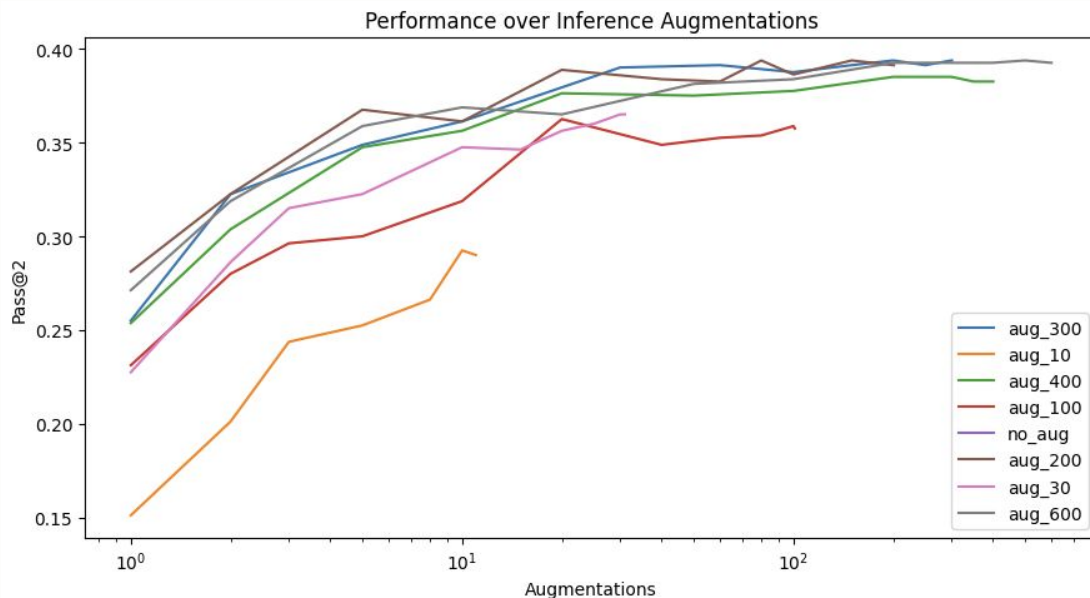
Data Augmentation

Vary # data augmentations

- Training: compile new dataset
- Inference: use random subsample of prediction pool for majority vote

Findings

- 1000 augmentations are not optimal
- 1 % of the data augmentation → ~75% of the effect
- Augmentations more important for training than for inference



Does the HRM Model Matter?

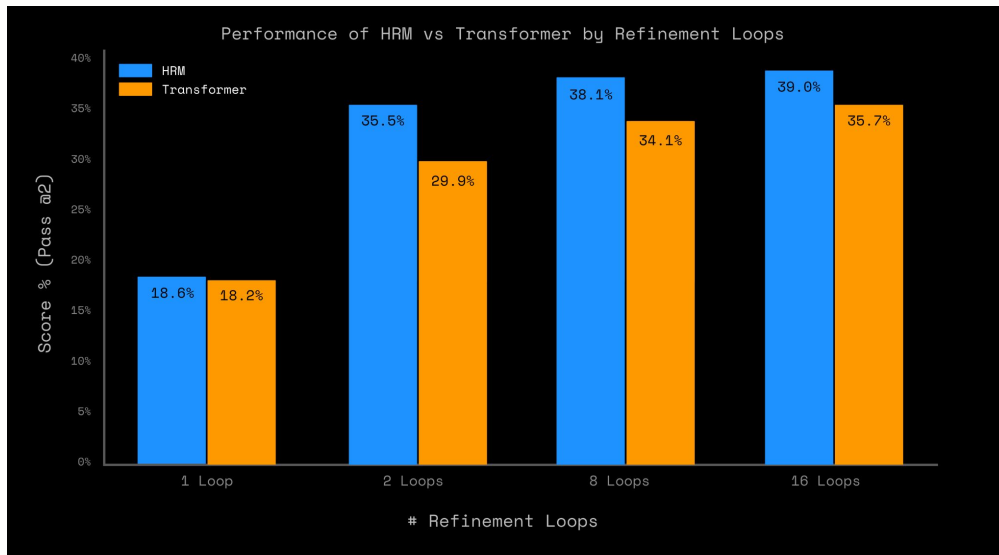
Compare HRM with regular transformer

- Matched parameter count
- Same hyperparameters
- Less (forward) compute

Findings

- Regular, unoptimized transformer is within ~5%.
- No refinement → matched
- More outer loop steps → gap closes

The model doesn't seem to matter, at least on ARC.

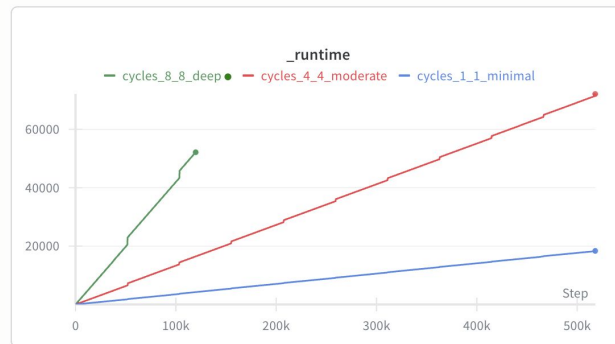
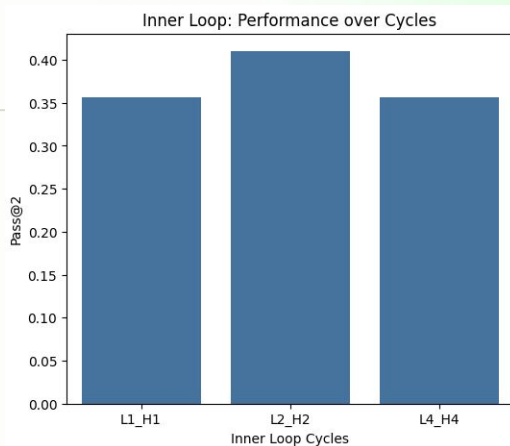


Does the Hierarchical Inner Loop Matter?

Vary L and H cycles

Findings

- 1-1 already achieves $\frac{7}{8}$ of the performance.
- Scaling higher does not improve further



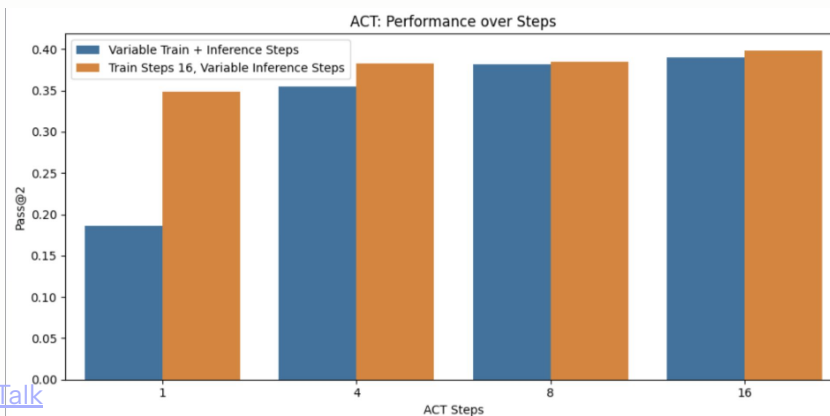
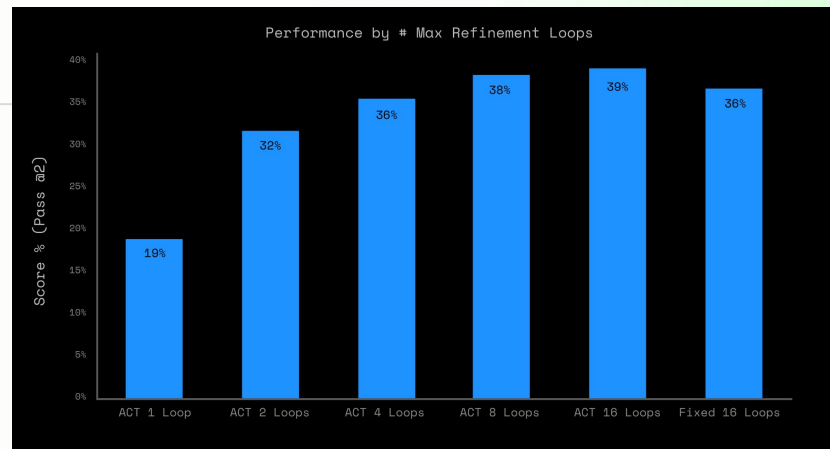
Does the outer refinement loop matter?

Vary max outer loop steps

Findings

- Refinement makes a big difference.
- Refinement at training > refinement at inference
- Learned halting signal leads works (only at training)

Refinement of predictions works!



Does HRM actually need that much data?

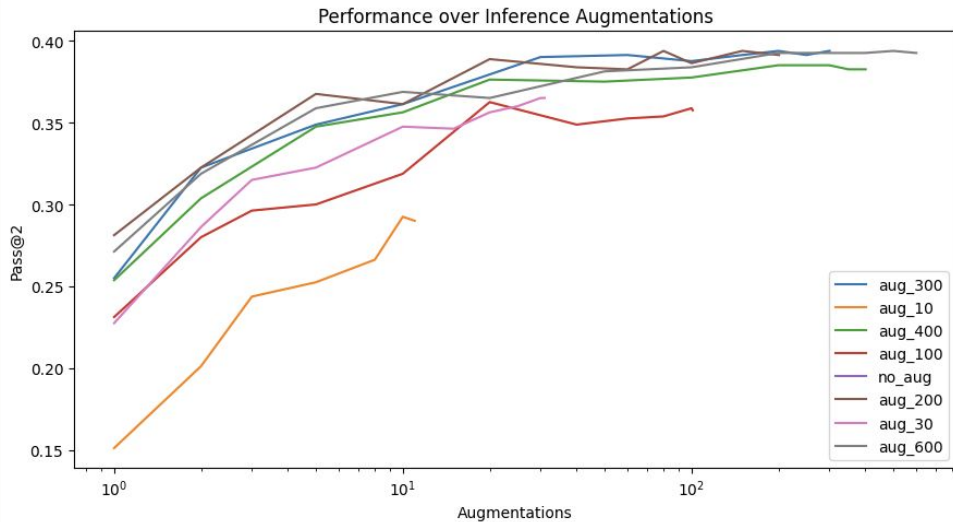
Vary # data augmentations

- Training: compile new dataset
- Inference: use random subsample of prediction pool for majority vote

Findings

- 1000 augmentations are not optimal
- 1 % of the data augmentation
→ ~75% of the effect
- Augmentations more important for training than for inference

Data augmentation helps, but a little is enough.



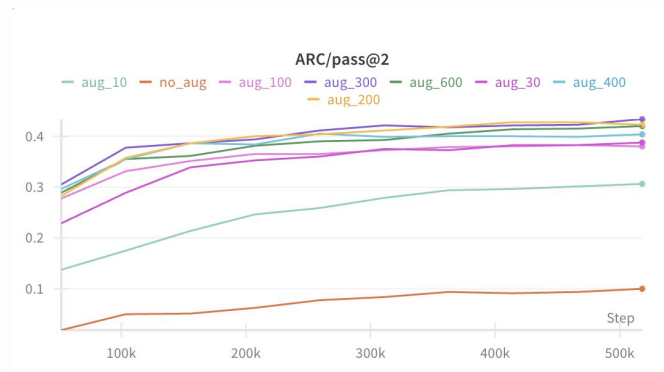
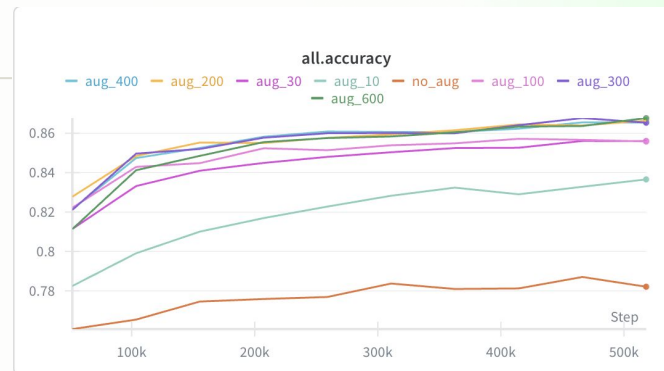
What's the impact of puzzle embeddings?

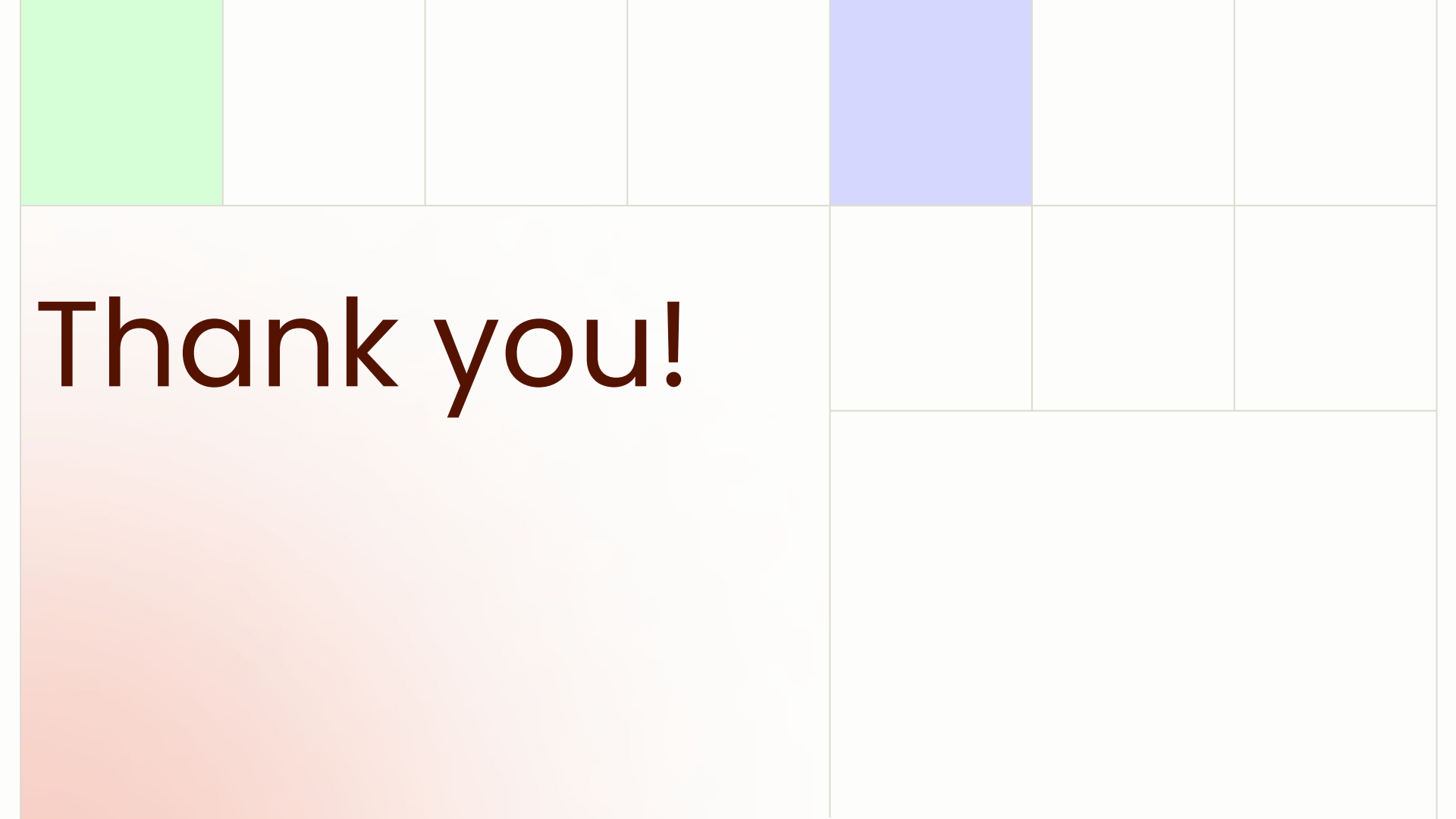
From the Authors

- Previous version of code without puzzle_id achieved ~76% per-token accuracy
- Related to existing run → <10% pass@2

Explanation Candidates

- Sequence length
- Condensed signal
- Easier task



The background features a grid of squares. The top row consists of five squares: the first is green, the second and third are light blue, the fourth is purple, and the fifth is light green. The bottom row consists of two large squares: the left one is light blue and contains the text 'Thank you!', and the right one is light green.

Thank you!

References

- HRM Paper
 - Codebase <https://github.com/sapientinc/HRM>
 - DeepWiki Page <https://deepwiki.com/sapientinc/HRM/1-overview>
- ARC Analysis <https://arcprize.org/blog/hrm-analysis>
 - Slides <https://docs.google.com/presentation/d/12IAuVKZXvbW6uCwzDhzN1PBh4fdjypjgyucYzoKJKMg/>